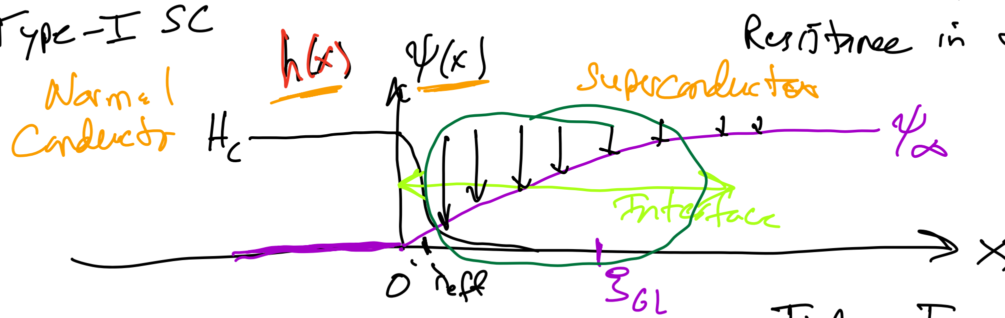
Lecture 16

GL Theory

→ Domain Wall Energies
Current in thin wires
Resistance in thin wires

Type-I SC

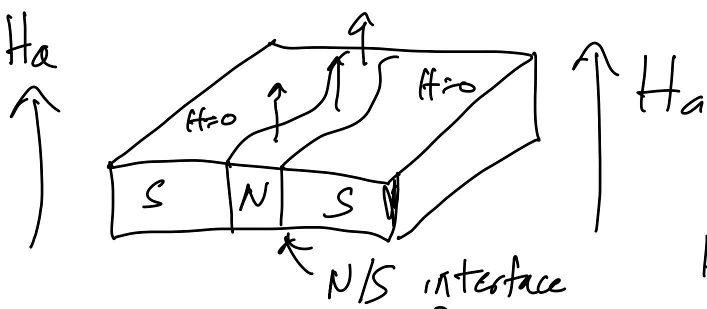


$\lambda_{eff}(T)$
$\xi_{GL}(T)$

Type-I $\kappa \ll 1$

$$\kappa = \frac{\lambda_{eff}}{\xi_{GL}}$$

$$\lambda_{eff} \ll \xi_{GL}$$

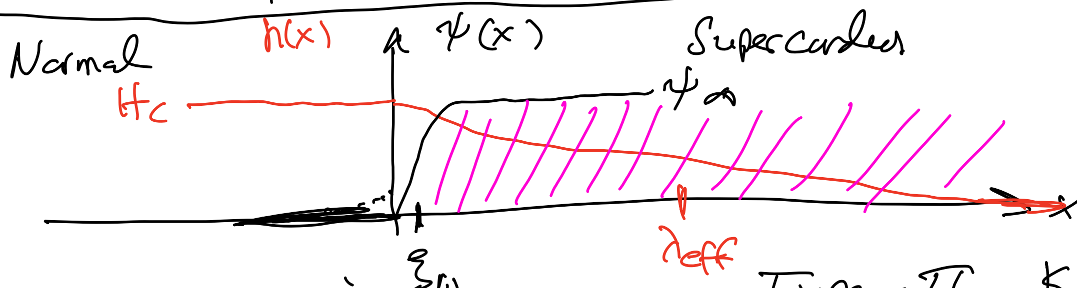


How much energy is converted in creating the domain wall.

positive energy $\mu_0 H^2/2$ positive price

negative energy $|\psi|^2 \rightarrow$ loosing

$$E_{DW}^{I} > 0$$



Type-II $\kappa \gg 1$

$$\lambda_{eff} \gg \xi_{GL}$$

Condensation energy is large and negative

positive contribution from magnetic screening is small.

$$E_{DW}^{II} < 0$$

Domain walls proliferate.
Proceeds until the quantization limit is achieved. $\Phi = \Phi_0 = h/2e$

Domain Wall Energy $\quad \gamma = \dfrac{\mu_0 H_c^2}{2} \delta \quad$ δ: width of the domain wall

$\hookrightarrow J/m^2$ Energy per unit area of a domain wall

Excess Gibbs free energy due to the domain wall

$$\delta = \int_{-\infty}^{\infty} \left[\left(1 - \frac{h(x)}{H_c}\right)^2 - \left(\frac{\psi(x)}{\psi_\infty}\right)^4 \right] dx$$

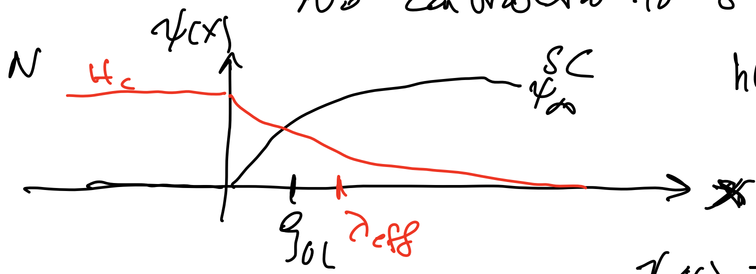
$h(x)$ = microscopic magnetic field

Normal Metal $h(x) \to H_c$ $\psi(x) \to 0$ $1 - \frac{h(x)}{H_c} = 0$ $(\psi(x)/\psi_\infty)^4 = 0$

No contribution to δ integral

SC $h(x) = 0$ $\psi(x)/\psi_\infty = 1$ $1 - 1 = 0$

No contribution to δ integral



$$h(x) = \begin{cases} H_c e^{-x/\lambda_{eff}} & x > 0 \\ H_c & x < 0 \end{cases}$$

$$\psi(x) = \begin{cases} \psi_\infty \left(1 - e^{-\sqrt{2}\, x/\xi_{GL}}\right) & x > 0 \\ 0 & x < 0 \end{cases}$$

$h(x)$, $\psi(x)$ not determined self-consistently

$$\Rightarrow \quad \delta = -\frac{3}{2}\lambda_{eff} + \frac{25}{24}\sqrt{2}\,\xi_{GL} \sim \xi_{GL} - \lambda_{eff}$$

Type I $K \ll 1$ $E^{I}_{DW} > 0$ $\delta \simeq +\frac{25}{24}\sqrt{2}\,\xi_{GL} = 1.47\,\xi_{GL}$

Type II $K \gg 1$ $\delta \simeq -1.5\,\lambda_{eff}$ $E^{II}_{DW} < 0$

Exact

$\frac{4\sqrt{2}}{3}\xi_{GL} = 1.9\,\xi_{GL}$

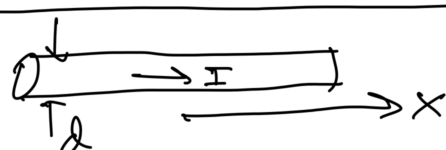
$-\frac{8}{3}(\sqrt{2} - 1)\lambda_{eff}$

$= -1.1\,\lambda_{eff}$

Boundary $\delta = 0$ $\Rightarrow$ $K = \frac{25\sqrt{2}}{36} = 0.98$

Exact Result $K = \frac{1}{\sqrt{2}} \simeq 0.7$

Current Flow in a Thin SC wire



Thin short wire

$d \ll \xi_{GL}$ $\psi(x)$ does not vary over the diameter of the wire

$\hookrightarrow$ Short compared to ξ_{GL} → No variation of $\psi(x)$ along X

$$\psi(x) = |\psi(x)| e^{i\phi(x)}$$

$\hookrightarrow$ $|\psi(x)|$ is a constant

$$\vec{J} = \frac{e^*}{m^*}|\psi|^2\left(\hbar\nabla\phi - e^*\vec{A}\right)$$

$$= e^*|\psi|^2\vec{V}_s$$

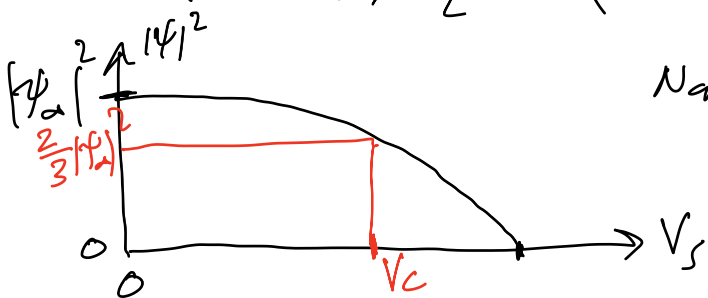
$$f_s - f_n = \alpha|\psi|^2 + \frac{\beta}{2}|\psi|^4 + \frac{1}{2}m^*V_s^2|\psi|^2 + \frac{\mu_0 H^2}{2}$$

$$\frac{\partial(f_s - f_n)}{\partial|\psi|} = 0 = 2\alpha|\psi| + 2\beta|\psi|^3 + m^* V_s^2|\psi| = 0$$

$$|\psi|^2 = -\frac{\alpha}{\beta}\left(1 - \frac{m^*}{2|\alpha|}V_s^2\right)$$

$$|\psi_\alpha|^2 = \frac{-\alpha}{\beta} \qquad \xi_{GL}^2 = \frac{\hbar^2}{2m^*|\alpha|}$$

$$|\psi|^2 = |\psi_\alpha|^2\left[1 - \left(\frac{m^*\xi_{GL}(T)}{\hbar}V_s\right)^2\right]$$



Nonlinear dependence of $|\psi|^2$ on V_s

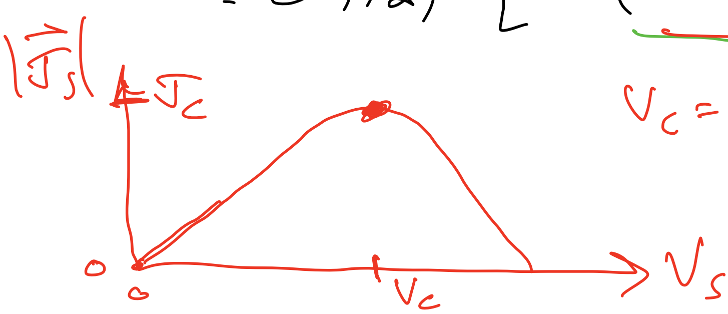
$$\vec{J}_s = e^*|\psi|^2\vec{V}_s$$

$$= e^*|\psi_\alpha|^2\left[1 - \left(\frac{m^*\xi_{GL}(T)V_s}{\hbar}\right)^2\right]\vec{V}_s$$

$e^{i\omega t}$

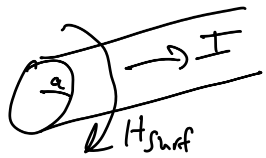
$\boxed{3\omega}$

V_c = critical velocity



$\frac{\partial J_s}{\partial v_s} = 0 \Rightarrow$ $\underline{v_c^2 = \frac{\hbar^2}{3(m^*)^2\xi_{0L}^2}}$, $\underline{J_c = \left(\frac{2}{3}\right)^{3/2}\frac{H_c}{\lambda_{eff}}}$

Silsbee's Rule



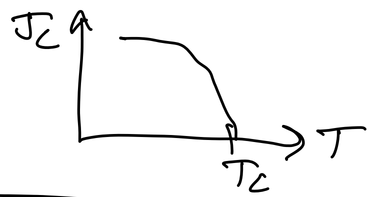
$H_{surf} = \frac{I}{2\pi a}$

$\Rightarrow H_c = \frac{I_c}{2\pi a}$

$J_c = \frac{I_c}{2\pi a \lambda_{eff}}$

$J_c = \frac{H_c(T)}{\lambda_{eff}(T)}$

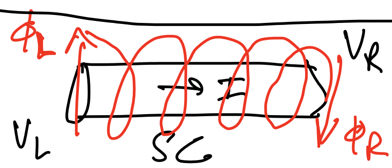
$J_c \sim (1-t)^{3/2}$ near T_c



How does resistance develop in a thin SC wire?

$\boxed{\vec{J}_s = \frac{e^*}{m^*}|\psi|^2(\hbar\nabla\phi)}$ $\psi = |\psi|e^{iqx}$

$V_L = V_R$



Phase helix has pitch $\frac{2\pi}{q}$

$v_s < v_c$

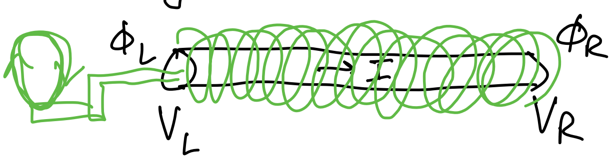
$J_s < J_c$

Fixed $\vec{J}_s$, fixed I

Fixed $\phi_R - \phi_L$

Zero resistance

To get resistance there has to be a voltage drop.



$V = V_R - V_L \neq 0$

Josephson Relation   AC Josephson Equation

$$\frac{d(\phi_R - \phi_L)}{dt} = \frac{2e}{\hbar}V$$

"Turn the crank" to add phase winding to the wire at a constant rate.

$V>0$, $\vec{E} \neq 0$ $\quad \vec{E} = \frac{\partial(\Lambda \vec{J}_s)}{\partial t} \Rightarrow \frac{dv_s}{dt} = \frac{eE}{m}$

$E \neq 0 \Rightarrow v_s$ must increase.

v_s is limited by v_c.

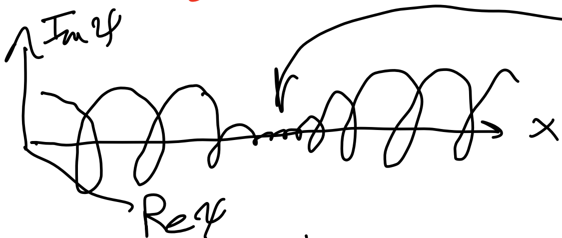
Dilemma as more phase windings are added.

$$J_s \sim |\psi(x)|^2 \frac{d\phi}{dx} = \text{constant} \propto I$$

↳ limited by v_c

↳ limited

Resolve the dilemma: Create a phase-slip center



Make $|\psi(x)| \to 0$ in some small region

Dissolve phase windings in units of 2π.

Fluctuation Phenomena

Heat Bath $\quad$ Borrow $k_B T$ of energy $\quad T \cong T_c$

$$\Delta F_0 = \frac{8\sqrt{2}}{3} \mu_0 H_c^2 (A \xi_{GL})$$

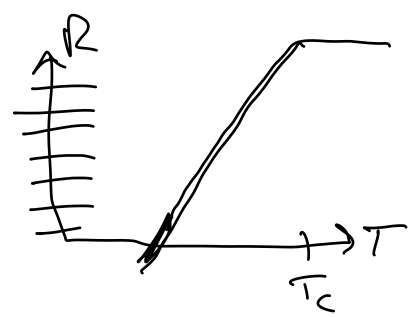
cost to create the fluctuation

cross sectional area of wire

$\Delta F_0 \to 0$ as $T \to T_c$

$\sim (1-t)^{3/2}$

$$R \sim \lim_{I \to 0} \frac{V}{I} = \frac{\pi \hbar^2 \Omega}{2e^2 k_B T} e^{-\Delta F_0 / k_B T}$$



LAMH

Langer, Ambegaokar, McCumber, Halperin